

# 5.1 Further Integration

## Question Paper

Course	CIEAS Maths
Section	5. Integration
Topic	5.1 Further Integration
Difficulty	Very Hard

**Time allowed:** 60  
**Score:** /46  
**Percentage:** /100

**Question 1**

Use calculus to find the value of

$$\int_2^6 \left( \frac{1}{x} + \frac{4}{2x} + \frac{3}{3x-2} \right) dx$$

giving your answer in the form  $p \ln p + q \ln q$ , where  $p$  and  $q$  are prime numbers to be found.

[5 marks]

**Question 2a**

(a) Find an expression for  $y$  given that

$$\frac{dy}{dx} = 5 \cos^2 4x \sin 4x$$

[3 marks]

**Question 2b**

(b) Integrate

$$\int 3x(5x^2 + 4)^4 dx$$

[3 marks]

**Question 3a**

(a) Show that

$$\left(\cos\left(\theta + \frac{\pi}{8}\right) + \sin\left(\theta + \frac{\pi}{8}\right)\right)\left(\sin\left(\theta + \frac{\pi}{8}\right) - \cos\left(\theta + \frac{\pi}{8}\right)\right) \equiv -\cos\left(2\theta + \frac{\pi}{4}\right)$$

[3 marks]

**Question 3b**

(b) Hence, or otherwise, find the exact value of

$$\int_0^{\frac{\pi}{8}} \left( \sin^2 \left( \theta + \frac{\pi}{8} \right) - \cos^2 \left( \theta + \frac{\pi}{8} \right) \right) d\theta$$

[3 marks]

**Question 4a**

(a) Show that

$$\int_0^1 e^{ax+b} dx = e^b \left( \frac{e^a - 1}{a} \right)$$

where  $a$  and  $b$  are constants, and  $a \neq 0$ .

[3 marks]

**Question 4b**

(b) Using your working from part (a), or otherwise, evaluate

$$\int_0^c e^{ax+b} dx$$

giving your answer in terms of  $a$ ,  $b$  and  $c$ , where  $a$ ,  $b$  and  $c$  are constants, and  $a \neq 0$ .

[2 marks]

**Question 5**

Find

$$\int (2 \tan x + 3 \sec x)^2 dx$$

[5 marks]

**Question 6a**

(a) Show that

$$\left(1 + \cot\left(2\theta + \frac{\pi}{4}\right)\right)\left(1 - \cot\left(2\theta + \frac{\pi}{4}\right)\right) \equiv 2 - \operatorname{cosec}^2\left(2\theta + \frac{\pi}{4}\right)$$

[3 marks]

**Question 6b**

(b) Hence, or otherwise, find an expression for  $f(\theta)$  given that

$$f'(\theta) = \left(2 + 2 \cot\left(2\theta + \frac{\pi}{4}\right)\right)\left(2 - 2 \cot\left(2\theta + \frac{\pi}{4}\right)\right)$$

[3 marks]

**Question 7**

Show that there are no positive values of  $a$  and  $b$  that satisfy the equation

$$\int_0^1 \frac{2a}{ax+b} dx = \int_b^{2a} \frac{1}{x} dx$$

[7 marks]

**Question 8**

Given that

$$f'(\theta) = \frac{9}{3 - 3 \sin^2 \left( 3\theta - \frac{\pi}{6} \right)}$$

and also that

$$\left[ f\left(\frac{\pi}{6}\right) \right]^2 - \left[ f\left(\frac{\pi}{9}\right) \right]^2 = \frac{8}{3}(1 - \sqrt{3})$$

find  $f(\theta)$ .

[6 marks]